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Response of Thermoelastic Interactions in Micropolar Porous Circular Plate with Three Phase Lag Model

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The present study deals with a homogeneous and isotopic micropolar porous thermoelastic circular plate by employing eigenvalue approach in the three phase lag theory of thermoelasticity due to thermomechanical sources. The expressions of components of displacements, microrotation, volume fraction field, temperature distribution, normal stress, shear stress and couple shear stress are obtained in the transformed domain by employing the Laplace and Hankel transforms. The resulting quantities are obtained in the physical domain by employing the numerical inversion technique. Numerical computations of the resulting quantities are made and presented graphically to show the effects of void, phase lags, relaxation time, with and without energy dissipation.

Keywords: micropolar porous, thermoelasticity, eigenvalue approach, Laplace and Hankel transforms, three phase lag model.

1. Introduction

The linear theory of micropolar thermoelasticity was developed by Nowacki [1] and Eringen [2]. Tauchert, Claus and Ariman [3] also examined the linear theory of micropolar coupled thermoelasticity. Dost and Tabarrok [4] represented the theory of generalized micropolar thermoelasticity taking into account Green Lindsay theory [5] of thermoelasticity. The theory of micropolar thermoelasticity including heat flux among constitutive variables was investigated by Chandrasekharaiah [6].

Ciarletta [7] established a solution of Galerkin type of a homogeneous and isotropic micropolar thermoelastic bodies and showed the effect of a concentrated heat source. Youssef [8] studied a problem of infinite body of generalized thermoelasticity with temperature dependent mechanical and thermal properties. Kumar and Gupta [9] discussed the wave propagation in a transversely isotropic micropolar thermoelastic solid without energy dissipation. Youssef [10] studied a problem of thermoelastic interactions in a isotropic medium with spherical cavity in the context of generalized theory of thermoelasticity with one relaxation time. Passarella and Zampoli [11] established the reciprocal and variational principles of convolutional type of micropolar thermoelastic solid having a centre of symmetry in the context of thermoelasticity theory of without energy dissipation. Sharma and Marin [12] investigated the effect of two temperatures on the reflection of plane waves in generalized thermoelastic solid. Kumar and Abbas [13] presented the two dimensional problem of micropolar thermoelastic solid with the effect of two temperatures in the context of Lord Shulman theory [14] due to thermal source.

Sharma and Bhargava [15] described the effect of thermal properties and stiffness on the amplitude ratios of reflected and refracted thermoelastic plane waves between the imperfect interface of generalized thermoelastic solid half space and thermal conducting viscous liquid. Kumar and Kumar [16] studied the deformation in micropolar thermoelastic diffusion solid subjected to inclined load with thermal laser pulse. Othman, Tantawi and Hilal [17] studied a two dimensional problem of isotropic and homogeneous initially stressed micropolar thermoelastic solid with microtemperatures and gravity field.

Iesan [18] studied the propagation of shock waves in micropolar elastic solids with voids. Marin [19, 20] investigated the basic equations in the theory of micropolar bodies with voids and formulated the initial boundary value problem by applying the general results from the theory of elliptic equations. Kumar and Choudhary [21, 22] presented the dynamical problem of a homogeneous and isotropic half space in micropolar elastic medium with voids due to a normal point source. Ciarletta, Svanadze and Buonanno [23] developed the linear theory of micropolar thermoelasticity of materials with voids and obtained some basic properties of wave numbers of the longitudinal and transverse plane harmonic waves. Othman and Lotfy [24] presented a model of the equations of a two dimensional problem in a micropolar generalized thermoelastic medium with voids due to various sources in the context of the Lord-Shulman [14], Green-Lindsay [5] theories and the classical dynamical couples theory. Lianngenga and Lalawmpuia [25] derived the constitutive equation and linear field equation in micropolar elastic materials containing voids. Kumar and Abbas [26] investigated a two dimensional axisymmetric problem in saturated porous media with incompressible fluid due to mechanical and thermal sources.

Roychoudhuri [27] investigated a three phase lag model by taking the heat conduction law that includes temperature gradient and thermal displacement gradient among constitutive variables in the theory of coupled thermoelasticity. This model is an extension of the thermoelastic models proposed by Lord Shulman [14] and Tzou [28, 29]. Kumar and Chawla [30] studied the propagation of longitudinal and transverse waves at the interface between uniform elastic solid half space and thermoelastic solid with three phase lag model. El-Karamany and Ezzat [31] established the uniqueness and reciprocal theorems and variational principle in the three phase lag micropolar thermoelasticity theory. Othman, Hasona and Abd-Elaziz [32] studied a two dimensional problem of micropolar thermoelastic isotropic medium with three phase lag model under the effect of rotation and initial stress. Kumar, Miglani and Rani [33] studied the axisymmetric problem of thick circular plate in a micropolar porous thermoelastic medium by employing eigenvalue approach subjected to thermomechanical sources. Marin, Agarwal and Codarcea [34] studied a mixed initial boundary value problem in three phase lag dipolar thermoelastic body.

In this paper, we investigate a homogeneous and isotropic micropolar porous thermoelastic circular plate with three phase lag model by employing eigenvalue approach. Laplace and Hankel transforms are used to obtain the expressions of components of displacements, microrotation, volume fraction field, temperature distribution and stresses in the transformed domain. The resulting quantities are obtained in the physical domain by employing the inversion technique of Laplace and Hankel transforms. The effects of void, phase lag, relaxation time, with and without energy dissipation have been shown on the resulting quantities and illustrated graphically.

2. Basic Equations

Following Kumar and Partap [35] and Roychoudhuri [27], the field equations and the constitutive relations in a micropolar porous thermoelastic medium with three phase lag model in the absence of body forces, body couples, heat sources and extrinsic equilibrated body force are taken as

$$(\lambda+\mu)\,\nabla\left(\nabla.\overrightarrow{u}\right)+(\mu+K)\,\nabla^2\overrightarrow{u}+K\nabla\times\overrightarrow{\phi}+b\nabla\phi^*-\nu\nabla T=\rho\frac{\partial^2\overrightarrow{u}}{\partial t^2}, \quad (1)$$

$$(\alpha+\beta+\gamma)\,\nabla\left(\nabla.\overrightarrow{\phi}\right)-\gamma\nabla\times\left(\nabla\times\overrightarrow{\phi}\right)+K\nabla\times\overrightarrow{u}-2K\overrightarrow{\phi}=\rho j\frac{\partial^2\overrightarrow{\phi}}{\partial t^2}, \quad (2)$$

$$\alpha_1\nabla^2\phi^*-b\left(\nabla.\overrightarrow{u}\right)-\xi_1\phi^*-\omega_0\frac{\partial\phi^*}{\partial t}+mT=\rho\chi\frac{\partial^2\phi^*}{\partial t^2}, \quad (3)$$

$$\left[K^*\left(1+\tau_\nu\frac{\partial}{\partial t}\right)+K_1^*\frac{\partial}{\partial t}\left(1+\tau_t\frac{\partial}{\partial t}\right)\right]\nabla^2T=$$
$$\left(1+\tau_q\frac{\partial}{\partial t}+\frac{\tau_q^2}{2}\frac{\partial^2}{\partial t^2}\right)\frac{\partial^2}{\partial t^2}\left[\rho C^*T+m\mathrm{T}_0\phi^*+\nu\mathrm{T}_0\left(\nabla.\overrightarrow{u}\right)\right], \quad (4)$$

$$t_{ij}=\lambda u_{r,r}\delta_{ij}+\mu\left(u_{i,j}+u_{j,i}\right)+K\left(u_{j,i}-\varepsilon_{ijk}\phi_k\right)-\nu T\delta_{ij}+b\delta_{ij}\phi^*, \quad (5)$$

$$m_{ij}=\alpha\phi_{r,r}\delta_{ij}+\beta\phi_{i,j}+\gamma\phi_{j,i}, \quad (6)$$

where $\overrightarrow{u}$, $\overrightarrow{\phi}$ are displacement and microrotation vector, ρ is the density, j is the micro inertia, λ, μ, K, α, β, γ are the micropolar constants, ϕ^* is the change in volume fraction field, α_1, b, ξ_1, ω_0, m and χ are the elastic constants due to the presence of voids, T is the change in temperature of the medium at any time, $\nu=(3\lambda+2\mu+K)\,\alpha_t$, α_t is the coefficient of linear thermal expansion, C^* is the specific heat at constant strain, K^* is the material characteristic, K_1^* is the coefficient of thermal conductivity, τ_t, τ_q and τ_v respectively, the phase lag of the temperature gradient, the phase lag of the heat flux and the phase lag of the thermal displacement. t_{ij}, m_{ij} and δ_{ij} are the stress tensor, couple stress tensor and Kronecker delta and

$$\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2} + \frac{\partial^2}{\partial z^2}$$

is the Laplacian operator.

3. Formulation of the Problem

A homogeneous and isotropic micropolar porous thermoelastic circular plate having thickness $2d$ is considered. The region $0 \le r \le \infty$, $-d \le z \le d$ is occupied by the plate. Cylindrical polar coordinate system $(r,\ \theta,\ z)$ is taken with symmetry about z-axis. We take a two dimensional problem and the middle surface of the plate is taken as the origin of the coordinates. The z-axis is normal to the plate along its thickness. The initial temperature T_0 is a constant temperature in the thick circular plate. The components of displacement vector $\overrightarrow{u}$ and microrotation vector $\overrightarrow{\phi}$ can be written for a two dimensional problem as

$$\overrightarrow{u} = (u_r, 0, u_z)\,, \quad \overrightarrow{\phi} = (0, \phi_\theta, 0)\ . \tag{7}$$

Introduce the following non-dimensional variables

$$r' = \frac{\omega^* r}{c_1}, z' = \frac{\omega^* z}{c_1}, u_r' = \frac{\rho c_1 \omega^* u_r}{\nu T_0}, u_z' = \frac{\rho c_1 \omega^* u_z}{\nu T_0}, \phi_\theta' = \frac{\rho c_1^2 \phi_\theta}{\nu T_0}, \phi^{*\prime} = \frac{\rho c_1^2 \phi^*}{\nu T_0},$$
$$T' = \frac{T}{T_0}, t' = \omega^* t, \tau_t' = \omega^* \tau_t, \tau_q' = \omega^* \tau_q, t_{ij}' = \frac{t_{ij}}{\nu T_0}, m_{ij}' = \frac{\omega^*}{c_1 \nu T_0} m_{ij}, \tag{8}$$

where

$$c_1^2 = \frac{\lambda + 2\mu + K}{\rho},$$

The Laplace and Hankel transforms are defined as follows

$$\overline{f}\,(r,z,s) = L\{\overline{f}\,(r,z,t)\} = \int_0^\infty f\,(r,z,t)\, e^{-st} dt, \tag{9}$$

$$\tilde{f}\,(\xi,z,s) = H\ \overline{f}\,(x,z,s)\ = \int_0^\infty r\overline{f}\,(x,z,s)\, J_n\,(\xi r)\, dr. \tag{10}$$

Equations (1)–(10), with the aid of (8)–(10), yield

$$\widetilde{\ddot{u}_r} = a_{11}\tilde{u}_r + a_{14}\widetilde{\phi *} + a_{15}\tilde{T} + b_{12}\widetilde{\dot{u}_z} + b_{13}\widetilde{\dot{\phi}_\theta}, \tag{11}$$

$$\widetilde{\ddot{u}_z} = a_{22}\tilde{u}_z + a_{23}\widetilde{\phi_\theta} + b_{21}\ \dot{\tilde{\psi}} + b_{24}\tilde{\dot{\phi}} + b_{25}\tilde{\dot{T}}, \tag{12}$$

$$\widetilde{\ddot{\phi}_\theta} = a_{32}\tilde{u}_z + a_{33}\widetilde{\phi_\theta} + b_{31}\widetilde{u_r'}, \tag{13}$$

$$\widetilde{\phi' *} = a_{41}\tilde{u}_r + a_{44}\widetilde{\phi *} + a_{45}\tilde{T} + b_{42}\widetilde{u_z'}, \tag{14}$$

$$\tilde{\ddot{T}} = a_{51}\tilde{u}_r + a_{54}\widetilde{\phi *} + a_{55}\tilde{T} + b_{52}\widetilde{u_z'}, \tag{15}$$

where

$$a_{11} = \left(\frac{\xi^2 + s^2}{\delta^2}\right), \quad a_{14} = \frac{p_0\xi}{\delta^2}, \quad a_{15} = -\frac{\xi}{\delta^2}, \quad a_{22} = \left(\xi^2\delta^2 + s^2\right), \quad a_{23} = p\xi,$$

$$a_{32} = -\xi\delta^{*2}, \quad a_{33} = \left(\xi^2 + \frac{s^2}{\delta_1^2} + 2\delta*^2\right),$$

$$a_{41} = p_0\delta *_1 \xi, \quad a_{44} = \left(\xi^2 + \delta *_3 s^2 + p_1\delta *_1 + \delta *_2 s\right),$$

$$a_{45} = -\overline{\nu}\delta_1^*, \quad a_{51} = \frac{\epsilon\xi s^2\left(1 + \tau_q s + \frac{\tau_q^2}{2}s^2\right)}{Z^*\left(1 + \tau_\nu s\right) + \left(1 + \tau_t s\right)s}, \quad a_{54} = \frac{\epsilon\overline{\nu}s^2\left(1 + \tau_q s + \frac{\tau_q^2}{2}s^2\right)}{Z^*\left(1 + \tau_\nu s\right) + \left(1 + \tau_t s\right)s},$$

$$a_{55} = \frac{\left(\xi^2(Z^*\left(1 + \tau_\nu s\right) + s\left(1 + \tau_t s\right)) + Q^* s^2\left(1 + \tau_q s + \frac{\tau_q^2}{2}s^2\right)\right)}{Z^*\left(1 + \tau_\nu s\right) + \left(1 + \tau_t s\right)s}, \quad b_{12} = \frac{\xi\left(1 - \delta^2\right)}{\delta^2},$$

$$b_{13} = \frac{p}{\delta^2}, \; b_{21} = -\xi\left(1 - \delta^2\right), \quad b_{24} = -p_0, \quad b_{25} = 1, \quad b_{31} = -\delta^{*2}, \quad b_{42} = p_0\delta_1^*,$$

$$b_{52} = \frac{\epsilon s^2\left(1 + \tau_q s + \frac{\tau_q^2}{2}s^2\right)}{Z^*\left(1 + \tau_\nu s\right) + \left(1 + \tau_t s\right)s}, \; c_2^2 = \frac{\mu + K}{\rho}, \; \delta^2 = \frac{c_2^2}{c_1^2},$$

$$p = \frac{K}{\rho c_1^2}, \; p_0 = \frac{b}{\rho c_1^2}, \delta^{*2} = \frac{K c_1^2}{\gamma\omega^{*2}},$$

$$\delta_1^2 = \frac{c_3^2}{c_1^2}, \; c_3^2 = \frac{\gamma}{\rho j}, \quad \delta_1^* = \frac{\rho c_1^4}{\alpha_1\omega^{*2}}, \quad \overline{\nu} = \frac{m}{\nu}, \quad p_1 = \frac{\xi_1}{\rho c_1^2}, \quad \delta_2^* = \frac{\omega_0 c_1^2}{\alpha_1\omega^*}, \; \delta_3^* = \frac{\rho\chi c_1^2}{\alpha_1},$$

$$Q^* = \frac{\rho C^* c_1^2}{K_1^*\omega^*}, \; \epsilon = \frac{\nu^2 T_0}{\rho K_1^*\omega^*}.$$

The system of equations (11)–(15) can be written as

$$\frac{d}{dz}W\left(\xi, \; z, \; s\right) = A\left(\xi, \; s\right)W\left(\xi, \; z, \; s\right), \tag{16}$$

where

$$W = \begin{bmatrix} U \\ DU \end{bmatrix}, \quad A = \begin{bmatrix} O & I \\ A_2 & A_1 \end{bmatrix}, \quad U = \begin{bmatrix} \tilde{u}_r \\ \tilde{u}_z \\ \widetilde{\phi}_\theta \\ \widetilde{\phi}^* \\ \tilde{T} \end{bmatrix}, \quad D = \frac{d}{dz},$$

$$O=\begin{bmatrix}0&0&0&0&0\\0&0&0&0&0\\0&0&0&0&0\\0&0&0&0&0\\0&0&0&0&0\end{bmatrix},\qquad I=\begin{bmatrix}1&0&0&0&0\\0&1&0&0&0\\0&0&1&0&0\\0&0&0&1&0\\0&0&0&0&1\end{bmatrix},$$

$$A_2=\begin{bmatrix}a_{11}&0&0&a_{14}&a_{15}\\0&a_{22}&a_{23}&0&0\\0&a_{32}&a_{33}&0&0\\a_{41}&0&0&a_{44}&a_{45}\\a_{51}&0&0&a_{54}&a_{55}\end{bmatrix},$$

$$A_1=\begin{bmatrix}0&b_{12}&b_{13}&0&0\\b_{21}&0&0&b_{24}&b_{25}\\b_{31}&0&0&0&0\\0&b_{42}&0&0&0\\0&b_{52}&0&0&0\end{bmatrix}.$$

The solution of (16) may be assumed

$$W(\xi,\ z,\ s)=X(\xi,\ s)\,e^{qz}, \tag{17}$$

which gives

$$A(\xi,\ s)\,W(\xi,\ z,\ s)=qW(\xi,\ z,\ s)\,. \tag{18}$$

Equation (18) leads to the eigenvalue problem. Therefore, the characteristic equation corresponding to the matrix $\mathbf{A}$ can be obtained by solving the following equation

$$\det(\mathbf{A}-q\mathbf{I})=0\,. \tag{19}$$

After solving the equation (19), we obtain

$$q^{10}-\lambda_1q^8+\lambda_2q^6-\lambda_3q^4+\lambda_4q^2-\lambda_5=0, \tag{20}$$

where $\lambda_1,\ \lambda_2,\ \lambda_3,\ \lambda_4$ and λ_5 are given in Appendix I.

We suppose that the roots of Eq. (20) are $\pm q_i,\ i=1,\ 2,\ 3,\ 4,\ 5$.

We determined the eigenvectors $X_i(\xi,\ s)$ corresponding to the eigenvalues q_i by solving

$$[\mathbf{A}-q\mathbf{I}]\,X_i(\xi,s)=0\,. \tag{21}$$

The set of eigenvector $X_i(\xi,\ s)$ may be written as

$$X_i(\xi,\ s)=\begin{bmatrix}X_{i1}(\xi,\ s)\\X_{i2}(\xi,\ s)\end{bmatrix}, \tag{22}$$

where

$$X_{i1}(\xi,\ s)=\begin{bmatrix}a_iq_i\\b_i\\-\xi\\d_i\\e_i\end{bmatrix},\qquad X_{i2}(\xi,\ s)=\begin{bmatrix}a_iq_i^2\\b_iq_i\\-\xi q_i\\d_iq_i\\e_iq_i\end{bmatrix},\qquad q=q_i;i=1,\ldots,5,$$

$$X_{j1}\left(\xi,\ s\right) = \begin{bmatrix} -a_i q_i \\ b_i \\ -\xi \\ d_i \\ e_i \end{bmatrix}, \quad X_{j2}\left(\xi,\ s\right) = \begin{bmatrix} a_i q_i^2 \\ -b_i q_i \\ \xi q_i \\ -d_i q_i \\ -e_i q_i \end{bmatrix},$$

$$j = i + 5,\ q = -q_i; i = 1, \ldots, 5\,,$$

where $a_i,\ b_i,\ d_i,\ e_i,\ \Delta_i,\ r_1,\ r_2,\ r_3$ and r_4 are given in Appendix II.

We assume the solution of Eq. (16) as

$$W\left(\xi,\ z,\ s\right) = \int_{i=1}^{5} N_i X_i\left(\xi,\ s\right)\cosh\left(q_i z\right)\,, \tag{23}$$

where $N_1,\ N_2,\ N_3,\ N_4$ and N_5 are arbitrary constants.

4. Boundary Conditions

The boundary conditions may be defined at the surface $z = \pm d$ of the plate as

1. A thermal source
$$\frac{dT}{dz} = \frac{F_0 \delta\left(t\right)\delta\left(r\right)}{2\pi r}, \tag{24}$$
2. A concentrated normal force
$$t_{zz} = \delta\left(t\right)\delta\left(a - r\right), \tag{25}$$
3. Vanishing of shear stress
$$t_{zr} = 0, \tag{26}$$
4. Vanishing of couple shear stress
$$m_{z\theta} = 0, \tag{27}$$
5. Vanishing of gradient of volume fraction field
$$\frac{d\phi^*}{dz} = 0, \tag{28}$$

where F_0 is the constant temperature applied on the boundary and $\delta()$ is the Dirac delta function.

The stress components t_{zz}, t_{zr} and $m_{z\theta}$ are given by

$$t_{zz} = \left(\lambda + 2\mu + K\right)\frac{\partial u_z}{\partial z} + \lambda\left(\frac{\partial u_r}{\partial r} + \frac{u_r}{r}\right) - \nu T + b\phi^*, \tag{29}$$

$$t_{zr} = \left(\mu + K\right)\frac{\partial u_r}{\partial z} + \mu\frac{\partial u_z}{\partial r} - K\phi_\theta, \tag{30}$$

$$m_{z\theta} = \gamma\frac{\partial \phi_\theta}{\partial z}. \tag{31}$$

The expressions of displacements, microrotation, volume fraction field, temperature distribution and stresses are obtained in the transformed domain by making the use of (5)–(10) and (23)–(31) as

$$\left(\tilde{u}_r, \tilde{u}_z, \widetilde{\phi_\theta}, \widetilde{\phi^*}, \tilde{T}\right) = \frac{1}{\Delta} \int_{i=1}^{5} (a_i q_i, b_i, -\xi, d_i, e_i)\Delta_i \cosh(q_i z) \tag{32}$$

$$\left(\widetilde{t_{zz}}, \ \widetilde{t_{zr}}, \ \widetilde{m_{z\theta}}\right) = \frac{1}{\Delta} \int_{i=1}^{5} (L_i, M_i, P_i)\, \Delta_i \cosh(q_i z)\,, \tag{33}$$

where

$$\Delta = \begin{vmatrix} S_1 & S_2 & S_3 & S_4 & S_5 \\ T_1 & T_2 & T_3 & T_4 & T_5 \\ U_1 & U_2 & U_3 & U_4 & U_5 \\ V_1 & V_2 & V_3 & V_4 & V_5 \\ W_1 & W_2 & W_3 & W_4 & W_5 \end{vmatrix},$$

and Δ_i, $(i = 1, 2, 3, 4, 5)$ are obtained from Δ by replacing i-th column of Δ with $|Q, R, 0, 0, 0|^{tr}$, also

$$S_i = e_i q_i \sinh(q_i d)\,, \quad T_i = L_i \cosh(q_i d)\,, \quad U_i = M_i \cosh(q_i d)\,, \quad V_i = P_i \cosh(q_i d)\,,$$

$$W_i = d_i q_i \sinh(q_i d)\,, \quad Q = \frac{F_0}{2\pi}, \quad R = aJ_0(\xi a)\,,$$

$$L_i = \left[\frac{\lambda \xi a_i q_i}{\rho c_1^2} + p_0 d_i - e_i + b_i q_i\right], \quad M_i = \left[-\frac{\mu \xi b_i}{\rho c_1^2} + \frac{\xi K}{\rho c_1^2} + \left(\frac{\mu + K}{\rho c_1^2}\right) a_i q_i^2\right],$$

$$P_i = \frac{-\gamma \xi \omega *^2 q_i}{\rho c_1^4}, \quad i = 1, \ldots 5.$$

5. Particular Cases

1. If we take $K^* = 0$ in Eqs. (32)–(33), then we obtain the corresponding results for micropolar porous thermoelastic with dual phase lag model.

2. If we take, $K^* = 0$, $\tau_t = \tau_q^2 = 0$ and $\tau_q = \tau_0$ in Eqs. (32)–(33), then we obtain the corresponding results for micropolar porous thermoelastic with one relaxation time.

3. If we take $\tau_\upsilon = K_1^* = \tau_q = \tau_q^2 = 0$ in Eqs. (32)–(33), then we obtain the corresponding results for micropolar porous thermoelastic with energy dissipation.

4. If we take $\tau_\upsilon = \tau_t = \tau_q = \tau_q^2 = 0$ in Eqs. (32)–(33), then we obtain the corresponding results for micropolar porous thermoelastic without energy dissipation.

5. If we neglect the porosity effect, i.e. α_1, b, ξ_1, ω_0, χ and ϕ^* tend to zero, we obtain the micropolar thermoelastic medium in which the boundary conditions take the form

$$\frac{dT}{dz} = \frac{F_0 \delta(t)\delta(r)}{2\pi r},$$
$$t_{zz} = \delta(t)\delta(a-r),$$
$$t_{zr} = 0,$$
$$m_{z\theta} = 0.$$

The corresponding expressions are given by

$$\left(\tilde{u}_r, \tilde{u}_z, \widetilde{\phi_\theta}, \widetilde{T}\right) = \frac{1}{\Delta^{**}} \int_{i=1}^{4} (a_i q_i, b_i, -\xi, e_i) \Delta_i^{**} \cosh(q_i z),$$

$$\left(\widetilde{t_{zz}}, \widetilde{t_{zr}}, \widetilde{m_{z\theta}}\right) = \frac{1}{\Delta^{**}} \int_{i=1}^{4} (L_i, M_i, P_i) \Delta_i^{**} \cosh(q_i z),$$

where

$$\Delta^{**} = \begin{vmatrix} S_1^{**} & S_2^{**} & S_3^{**} & S_4^{**} \\ T_1^{**} & T_2^{**} & T_3^{**} & T_4^{**} \\ U_1^{**} & U_2^{**} & U_3^{**} & U_4^{**} \\ V_1^{**} & V_2^{**} & V_3^{**} & V_4^{**} \end{vmatrix},$$

and Δ_i^{**} $(i = 1,2,3,4)$ are obtained from Δ^{**} by replacing i-th column of Δ^{**} with $|Q, R, 0, 0|^{tr}$
also

$$S_i^{**} = e_i q_i \sinh(q_i d), T_i^{**} = L_i^{**} \cosh(q_i d), U_i^{**} = M_i^{**} \cosh(q_i d),$$
$$V_i^{**} = P_i^{**} \cosh(q_i d), \quad i = 1,2,3,4$$
$$L_i^{**} = \left[\frac{\lambda \xi a_i q_i}{\rho c_1^2} - e_i + b_i q_i\right], M_i^{**} = \left[-\frac{\mu \xi b_i}{\rho c_1^2} + \frac{\xi K}{\rho c_1^2} + \left(\frac{\mu + K}{\rho c_1^2}\right) a_i q_i^2\right], \quad i = 1,2,3,4$$
$$P_i^{**} = \frac{-\gamma \xi {\omega^*}^2 q_i}{\rho c_1^4}, \quad i = 1,2,3,4.$$

6. Inversion of Transforms

The transformed displacements, microrotation, volume fraction field, temperature distribution and stresses are of the form $\tilde{f}(\xi, z, s)$ and we have to obtain the function $f(r, z, t)$, the inversion formula for the Hankel transform is taken as

$$\tilde{f}(\xi, z, s) = \int_0^\infty \xi \overline{f}(\xi, z, s) J_n(\xi r) d\xi. \tag{34}$$

The inversion formula for the Laplace transforms is taken as

$$f(r, z, t) = \frac{1}{2\iota\pi} \int_{c-\iota\infty}^{c+\iota\infty} \overline{f}(r, z, s) e^{-st} ds, \tag{35}$$

where c is an arbitrary constant greater than all real parts of the singularities of $\overline{f}(r, z, t)$.

7. Numerical Results and Discussions

Following Eringen [36] gives the values for micropolar parameters as

$$\lambda = 9.4 \times 10^{10}\,\mathrm{Nm}^{-2}, \quad \mu = 4.0 \times 10^{10}\,\mathrm{Nm}^{-2}, \quad K = 1.0 \times 10^{10}\,\mathrm{Nm}^{-2},$$

$$\rho = 1.74 \times 10^{3}\,\mathrm{Kgm}^{-3}, \quad j = 0.2 \times 10^{-19}\,\mathrm{m}^{2}, \quad \gamma = 0.779 \times 10^{-9}\,\mathrm{N},$$

$$F_0 = 1.$$

Following Dhaliwal and Singh [37] give the values for thermal parameters as

$$C^{*} = 1.04{\times}10^{3}\,\mathrm{JKg}^{-1}\,\mathrm{K}^{-1}, \quad K_1^{*} = 1.7{\times}10^{6}\,\mathrm{Jm}^{-1}\mathrm{s}^{-1}\,\mathrm{K}^{-1}, \quad \alpha_t = 2.33{\times}10^{-5}\,\mathrm{K}^{-1},$$

$$\tau_t = 0.1 \times 10^{-13}\,\mathrm{s}, \quad \tau_q = 0.2 \times 10^{-13}\,\mathrm{s}, \quad \tau_0 = 6.131 \times 10^{-13}\,\mathrm{s},$$

$$\tau_\nu = 8.765 \times 10^{-13}\,\mathrm{s}, \quad T_0 = 0.298 \times 10^{3}\,\mathrm{K}, \quad m = 1.13849 \times 10^{10}\,\mathrm{N/m^2\,K}.$$

The values of void parameters are

$$\alpha_1 = 3.688 \times 10^{-9}\,\mathrm{N}, \quad b = 1.138494 \times 10^{10}\,\mathrm{N/m^2}, \quad \xi_1 = 1.1475 \times 10^{10}\,\mathrm{N/m^2},$$

$$\chi = 1.1753 \times 10^{-19}\,\mathrm{m}^2, \quad \omega_0 = 0.0787 \times 10^{-1}\,\mathrm{N} \times \mathrm{s/m^2}.$$

Figures 1–5 represent the variations of normal stress, shear stress, couple shear

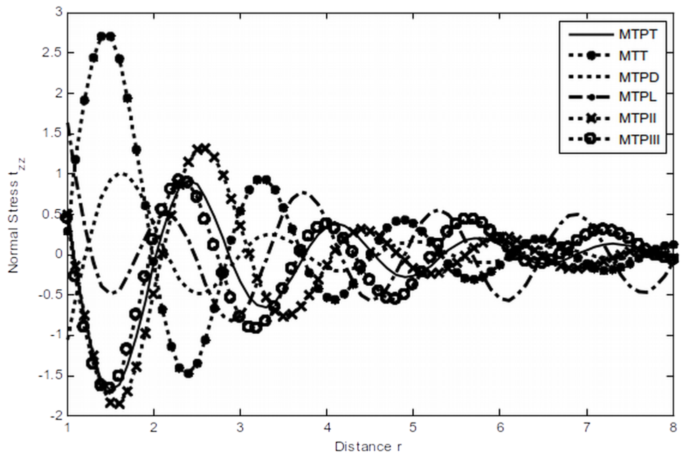

Figure 1 Variations of normal stress t_{zz}

stress, volume fraction field and temperature distribution with distance r in case of micropolar thermoelastic porous with three phase lag model (MTPT), micropolar thermoelastic with three phase lag model (MTT), micropolar thermoelastic porous with dual phase lag model (MTPD), micropolar thermoelastic porous with Lord Shulman theory (MTPL), micropolar thermoelastic porous with GN type II (MT-PII) and micropolar thermoelastic porous with GN type III (MTPIII). In all these figures, MTPT, MTPD, MTPL, MTT, MTPII and MTPIII corresponding to solid line (——), small dash line (- - - - -), dash line (– – – -), dash line with centred symbol (-∗-∗-∗-), dash line with centred symbol (- × - ×) and dash line with centred symbol (- o - o -) respectively.

Figure 1 shows that the values of t_{zz} initially decrease for $1 \leq r \leq 1.6$ for MTPT, MTPL, MTPII and MTPIII and increase for $1 \leq r \leq 1.5$ for MTT and MTPD and then oscillate as r increases. Near and away from the source, the values for MTPT, MTPII and MTPIII are similar. t_{zz} has maximum value $1.3 \leq r \leq 1.7$ for MTT and minimum value for $1.6 \leq r \leq 1.8$ for MTPII.

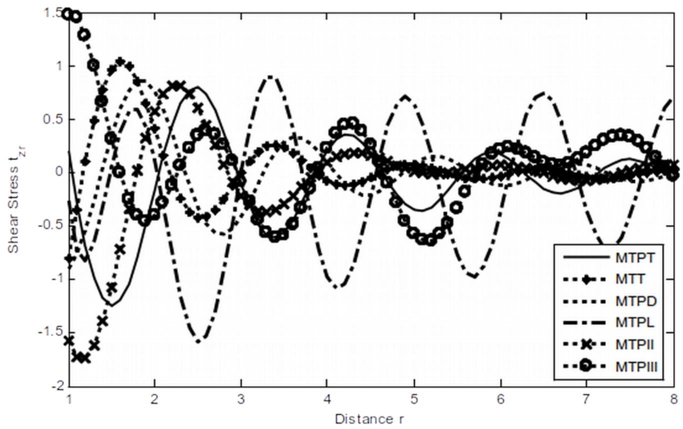

Figure 2 Variations of shear stress t_{zr}

Figure 2 illustrates that the values of t_{zr} initially decrease for MTPT, MTPL, MTPII and MTPIII for the ranges $1 \leq r \leq 1.5$, $1 \leq r \leq 1.2$, $1 \leq r \leq 1.3$ and $1 \leq r \leq 2$ respectively and increase for MTT and MTPD for $1 \leq r \leq 1.6$ and $1 \leq r \leq 1.8$ respectively. t_{zr} initially attains the maximum value for MTPIII and minimum value for MTPII. All the curves oscillate with different amplitude. Away from the source, MTPT, MTT, MTPD, MTPII and MTPIII have similar behavior.

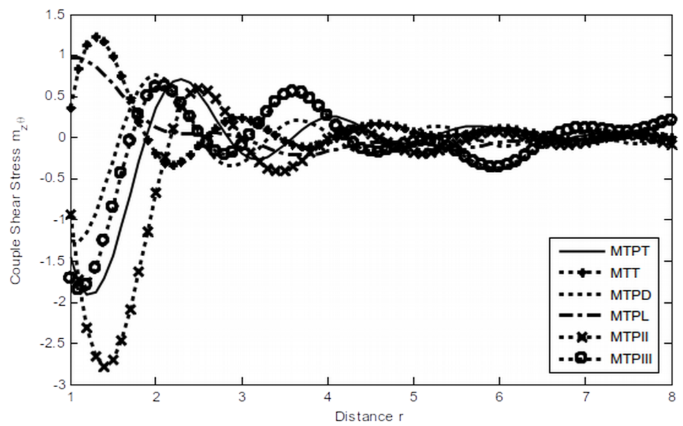

Figure 3 Variations of couple shear stress $m_{z\theta}$

Figure 3 exhibits that the values of $m_{z\theta}$ initially decrease for MTPT, MTPD, MTPL, MTPII and MTPIII and increase for MTT and then oscillate about the origin. $m_{z\theta}$ has similar values for MTPT, MTT, MTPD, MTPL and MTPII away from the source. The value of $m_{z\theta}$ for MTT is large for $1.2 \leq r \leq 1.6$ and small for MTPII for $1.2 \leq r \leq 1.8$ in comparison to other cases.

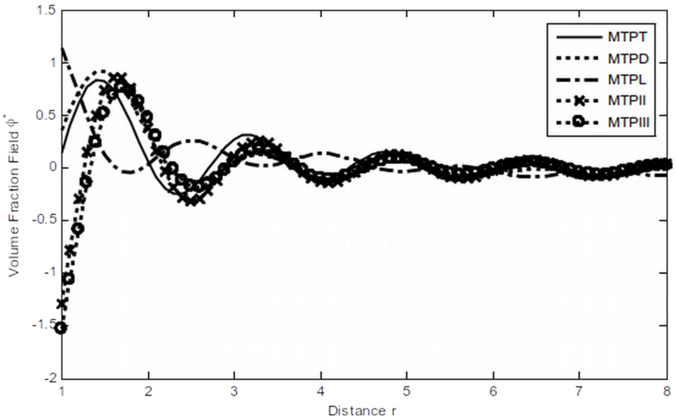

Figure 4 Variations of volume fraction field ϕ^*

Figure 4 indicates that the values of ϕ^* increase for MTPT, MTPD, MTPII and MTPIII for $1 \leq r \leq 1.5$ and $1 \leq r \leq 1.8$ respectively and then oscillate about the origin as r increases. The value of ϕ^* decreases for $1 \leq r \leq 2$ and then oscillates with large amplitude. ϕ^* has similar values for $3.5 \leq r \leq 8$ as in the cases of MTPT, MTPD, MTPII and MTPIII. At the beginning, ϕ^* has large value for MTPL and small values for MTPII and MTPIII. Near the application of the source, ϕ^* has same value for MTPII and MTPIII and away from the source, MTPT, MTPD, MTPII and MTPIII have the same values.

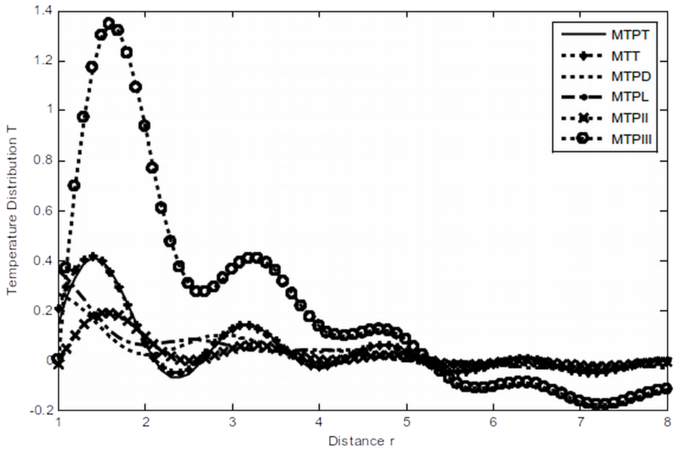

Figure 5 Variations of temperature distribution T

Figure 5 shows that the values for T initially increase for MTPT, MTT, MTPII and MTPIII for $1 \leq r \leq 1.4$, $1 \leq r \leq 1.6$ respectively and decrease for $1 \leq r \leq 2$ for MTPD and MTPL. The values of T coincide for MTPT and MTT for $1 \leq r \leq 8$. T has the minimum value for MTPII and MTPIII in comparison to the other cases near the application of the source. T attains its large value for $1.3 \leq r \leq 2$ and small value for $5.6 \leq r \leq 8$ for MTPIII. Away from the source, T has similar value for MTPT, MTT, MTPD, MTPL and MTPII.

8. Conclusions

In this paper, an axisymmetric problem for three phase lag micropolar porous thermoelastic circular plate by employing eigenvalue approach subjected to thermomechanical sources has been investigated. Integral transform technique has been applied to solve the problem. All the resulting quantities are influence by the miropolarity, void and thermal effect. Effects of void, dual phase lag, thermal relaxation time, with and without energy dissipation are presented on the normal stress, shear stress, couple shear stress, volume fraction field and temperature distribution. A significant oscillatory behavior is observed on the resulting quantities. The resulting quantities have similar behavior for MTPII and MTPIII theories. It is also observed that the temperature distribution has similar behavior for MTPT, MTT, MTPII and MTPIII which is opposite to MTPD and MTPL. It is also evident that near the application of the source, normal stress and shear stress have similar behavior and variation for MTPT, MTPII and MTPIII and opposite to those MTT and MTPD.

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Appendix I

$$\lambda_1 = -(a_{11} + a_{22} + a_{33} + a_{44} + a_{55} + b_{12}b_{21} + b_{13}b_{31} + b_{25}b_{52} + b_{24}b_{42}),$$

$$\begin{gathered}
\lambda_2 = -a_{14}a_{41} + a_{33}a_{55} + a_{44}a_{55} + a_{11}a_{55} + a_{22}a_{55} + a_{33}a_{44} + a_{11}a_{33} + a_{22}a_{33} \\
+a_{11}a_{44} + a_{22}a_{44} + a_{11}a_{22} - a_{15}a_{51} - a_{45}a_{54} - a_{23}a_{32} + (a_{33} + a_{44} + a_{55})\, b_{12}b_{21} \\
-(a_{14}b_{42} + a_{15}b_{52} + a_{32}b_{13})\, b_{21} + (a_{11} + a_{33} + a_{55})\, b_{42}b_{24} + (a_{11} + a_{33} + a_{44})\, b_{25}b_{52} \\
-(a_{41}b_{24} + a_{23}b_{31} + a_{51}b_{25})\, b_{12} + (a_{22} + a_{44} + a_{55} + b_{42}b_{24} + b_{25}b_{52})\, b_{31}b_{13} \\
-a_{45}b_{52}b_{24} - a_{54}b_{42}b_{25},
\end{gathered}$$

$$\begin{gathered}
\lambda_3 = (a_{11}a_{22} + a_{22}a_{55})(a_{33} + a_{44}) - a_{23}a_{32}\,(a_{11} + a_{44} + a_{55}) \\
+a_{11}a_{55}\,(a_{22} + a_{33} + a_{44}) + a_{33}a_{44}\,(a_{11} + a_{22} + a_{55}) - a_{45}a_{54}\,(a_{11} + a_{22} + a_{33}) \\
-a_{14}a_{41}\,(a_{22} + a_{33} + a_{55}) - a_{15}a_{51}\,(a_{22} + a_{33} + a_{44}) + b_{42}b_{25} \\
(a_{14}a_{51} - a_{11}a_{54} - a_{33}a_{54}) + b_{52}b_{25}(-a_{14}a_{41} + a_{11}a_{33} + a_{14}a_{44} + a_{33}a_{44}) \\
+b_{52}b_{24}\,(a_{15}a_{41} - a_{11}a_{45} - a_{33}a_{45}) - b_{12}b_{25}(a_{33}a_{51} + a_{44}a_{51} - a_{41}a_{54}) \\
-b_{12}b_{24}(a_{33}a_{41} - a_{45}a_{51} + a_{41}a_{55}) + b_{42}b_{24}(-a_{15}a_{51} + a_{11}a_{33} + a_{11}a_{55} + a_{33}a_{55}) \\
-a_{32}b_{21}b_{13}\,(a_{44} + a_{55}) + b_{31}b_{13}(a_{22}a_{44} + a_{22}a_{55} + a_{44}a_{55} - a_{45}a_{54}) \\
+b_{21}b_{42}(a_{15}a_{54} - a_{14}a_{33} - a_{14}a_{55}) + b_{21}b_{52}(a_{14}a_{45} - a_{15}a_{33} - a_{15}a_{44}) \\
+b_{12}b_{21}\,(-a_{45}a_{54} + a_{33}a_{44} + a_{44}a_{55} + a_{33}a_{55}) - b_{12}b_{31}\,(a_{23}a_{44} + a_{23}a_{55}) \\
-b_{31}b_{13}\,(a_{54}b_{42}b_{25} - a_{44}b_{52}b_{25} + a_{45}b_{52}b_{24} - a_{55}b_{24}b_{42}) + a_{32}b_{13} \\
(a_{51}b_{25} + a_{41}b_{24}) + a_{15}a_{41}a_{54} + a_{14}a_{45}a_{51} + (a_{14}b_{42} + a_{15}b_{52})\, a_{23}b_{31},
\end{gathered}$$

$$\begin{gathered}
\lambda_4 = (a_{44}a_{55} - a_{45}a_{54})\,(a_{33}b_{12}b_{21} - a_{23}b_{12}b_{31}) + (a_{15}a_{54} - a_{14}a_{55}) \\
(a_{33}b_{21}b_{42} - a_{23}b_{31}b_{42}) + (a_{14}a_{45} - a_{15}a_{44})\,(a_{33}b_{21}b_{52} - a_{23}b_{31}b_{52}) \\
+(a_{45}a_{54} - a_{44}a_{55})\,(a_{32}b_{21}b_{13} - a_{22}b_{31}b_{13}) + a_{33}b_{12}b_{24}\,(a_{45}a_{51} - a_{41}a_{55})
\end{gathered}$$

$$+a_{33}b_{42}b_{24}(-a_{15}a_{51}+a_{11}a_{55})+a_{32}b_{13}b_{24}(a_{41}a_{55}-a_{45}a_{51})$$
$$+(a_{41}a_{54}-a_{44}a_{51})(a_{33}b_{12}b_{25}-a_{32}b_{13}b_{25})+a_{33}b_{42}b_{25}(a_{14}a_{51}-a_{11}a_{54})$$
$$+a_{33}b_{52}b_{25}(a_{11}a_{44}-a_{14}a_{41})+a_{15}a_{51}(a_{23}a_{32}-a_{22}a_{33}-a_{22}a_{44}-a_{33}a_{44})$$
$$+a_{14}a_{45}a_{51}(a_{22}+a_{33})+a_{15}a_{41}a_{54}(a_{22}+a_{33})+a_{45}a_{54}(a_{23}a_{32}-a_{11}a_{22}-a_{11}a_{33}$$
$$-a_{22}a_{33})+a_{14}a_{41}(a_{23}a_{32}-a_{22}a_{33}-a_{22}a_{55}-a_{33}a_{55})-a_{11}a_{23}a_{32}(a_{44}+a_{55})$$
$$+a_{55}(a_{11}a_{22}a_{33}-a_{23}a_{32}a_{44})+a_{11}a_{22}a_{44}(a_{33}+a_{55})+a_{33}a_{44}a_{55}(a_{11}+a_{22})$$
$$+(a_{15}a_{41}-a_{11}a_{45})a_{33}b_{24}b_{52},$$
$$\lambda_5=(a_{22}a_{33}-a_{23}a_{32})(a_{11}a_{44}a_{55}-a_{11}a_{45}a_{54}+a_{14}a_{45}a_{51})$$
$$+(a_{15}a_{54}-a_{14}a_{55})(a_{22}a_{33}a_{41}-a_{23}a_{32}a_{41})+(a_{23}a_{32}-a_{22}a_{33})a_{15}a_{44}a_{51}.$$

Appendix II

$$a_i=\frac{\xi}{\Delta_i}[r_1^2\, r_2\left(r_3\left(1-\delta^2\right)+p\delta^{*2}\right)-p_0^2\delta_1^* r_3$$
$$+\epsilon s^2 r_1 r_5\{r_3\left(r_2+\overline{\nu}^2\delta_1^*\left(1-\delta^2\right)-2\overline{\nu}p_0\delta_1^*-\overline{\nu}\delta_1^*\right)+p\overline{\nu}^2\delta_1^*\delta^{*2}\}],$$
$$b_i=\frac{-1}{\Delta_i}[r_1^2\{r_2\left(r_3r_4+p\delta^{*2}q_i^2\right)-p_0^2\delta_1^*\xi^2 r_3\}$$
$$+\epsilon s^2 r_3 r_1 r_5\left(\xi^2\ r_2+\overline{\nu}^2\delta_1^* r_4-2\overline{\nu}p_0\delta_1^*\xi^2\right)+\epsilon s^2 p\overline{\nu}^2\delta_1^*\delta^{*2}q_i^2 r_1 r_5],$$
$$d_i=\delta_1^*\left(p_0 r_1+\epsilon s^2\overline{\nu}r_5\right)\left(\xi a_i+b_i\right)q_i/\left(-r_2r_1-\epsilon s^2\overline{\nu}^2\delta_1^* r_5\right),$$
$$e_i=[\epsilon r_5\{s^2\left(r_2r_1+\epsilon s^2\overline{\nu}^2\delta_1^* r_5\right)$$
$$-\overline{\nu}\delta_1^*\left(p_0r_1+\epsilon\overline{\nu}r_5\right)\}]\left(\xi a_i+b_i\right)q_i/\{r_1\left(-r_2r_1-\epsilon s^2\overline{\nu}^2\delta_1^* r_5\right)\},$$
$$\Delta_i=\delta^{*2}[r_1^2\, r_2\left(\xi^2+s^2-q_i^2\right)+p_0^2\delta_1^*\left(q_i^2-\xi^2\right)$$
$$+\epsilon s^2 r_2 r_1 r_5\left(\xi^2-q_i^2\right)+\epsilon s^2\delta_1^* r_1 r_5\{\overline{\nu}^2\left(\xi^2+s^2-q_i^2\right)-2p_0\overline{\nu}\left(\xi^2-q_i^2\right)\}],$$
$$r_1=\left(\left(\xi^2(Z^*\left(1+\tau_\nu s\right)+s\left(1+\tau_t s\right))+Q^* s^2\left(1+\tau_q s+\frac{\tau_q^2}{2}s^2\right)\right)+\right.$$
$$\left.-q_i^2(Z^*\left(1+\tau_\nu s\right)+\left(1+\tau_t s\right)s)\right),$$
$$r_2=\left(\xi^2+\delta_3^* s^2+p_1\delta_1^*+\delta_2^* s-q_i^2\right),\quad r_3=\left(\xi^2+\frac{s^2}{\delta_1^2}+2\delta^{*2}-q_i^2\right),$$
$$r_4=\left(\xi^2+s^2-\delta^2 q_i^2\right),\quad r_5=\left(1+\tau_q s+\frac{\tau_q^2}{2}s^2\right).$$